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Towards a critical edition of Fibonacci's *Liber Abaci*

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by Concetta Carotenuto

1. Introduction: the exegetical and textual questions of the *Liber Abaci*

The critical edition of Fibonacci's *Liber Abaci* must be based on the collation of the nine witnesses who have passed the treatise in its entirety and the other eight that preserve the chapters on the so-called "practical issues"¹.

Before presenting some examples of emendations to the text of the *Liber's vulgata*, it should be stated that investigating the relations among the witnesses of a technical text – especially when, as in this case, it consists of a treatise on mathematics – presents some peculiarities that can be summarized as follows:

- Errors due to distraction for the repetitive nature of the treatise;
- Errors due to ignorance of the copyist who may not be an expert on the subject;
- Variations due to synthesis or rework because a work of technical nature is devoid of that golden sacred style that requires the copyist to respect the form of the original expression as well as the content;

¹ For the full list of the Fibonacci treatise's witnesses cfr., in this issue G. Germano, *Editorial New Perspectives on Fibonacci's Liber Abaci*. For the reader's convenience, I carry the classification adopted for the manuscripts, since I will refer to them on the basis of this classification. *A*: Milano, Biblioteca Ambrosiana, ms. I. 72 Sup., ff. 1-124, parchment, 13th century; *S*: Siena, Biblioteca Comunale degli Intronati, ms. L. IV. 20, ff. 1-224, parchment, ending 13th - early 14th century; *V*: Città del Vaticano, Biblioteca Apostolica Vaticana, ms. Palat. Lat. 1343, ff. 1-174, parchment, ending 13th - early 14th century; *F*: Firenze, Biblioteca Nazionale Centrale, ms. Conv. Soppr. C. 1. 2616, ff. 1-214, parchment, early 14th century; *F*₁: Firenze, Biblioteca Nazionale Centrale, ms. Magl. XI. 21, ff. 1-285, parchment, 14th century; *R*: Firenze, Biblioteca Riccardiana, ms. 783, ff. 1-346, paper, 15th century; *F*₂: Firenze, Biblioteca Nazionale Centrale, ms. Fond. Prin. II. III. 25 [Magl. XI. 22], ff. 1-175, paper, 15th century or 16th century; *N*: Napoli, Biblioteca Nazionale, ms. VIII. C. 18, ff. 1-285, paper, 17th century; *B*: Berlin, Staatsbibliothek, Preußischer Kulturbesitz, ms. Lat. Fol. 418, ff. 1-805, paper, 17th century.

- Variations due to the intervention of an “acculturated” copyist that corrects errors in mathematical calculations when he finds them.

In the case of Fibonacci's *Liber Abaci* the collation of manuscripts brought to light an enormous amount of variations, which cover in particular the Arabic numerals.

The merit of Fibonacci lies in introducing systematically the Hindu-Arabic numeric system in Europe: it was a novelty and copyists were not accustomed to its use. Over time, moreover, the signs indicating the Arabic numerals have changed (in particular the 7, 4 and 5) and this could generate misunderstandings in copiers. Moreover, in a fast writing, the signs for the 1 and 2, but also for the 2 and 3, or 0, 6 and 9, tend to merge². This means, in my view, that such errors, at least in the first instance, should not be regarded as significant because, for example, N – which is an acculturated copyist – can correct an error in its own antigraph restoring the original lection. Other variants relate to the recurring oscillation between *virga* and *virgula* (synonymic terms that indicate the “fraction line”) that appear to be random, even within a single manuscript. In principle, in each case, A seems to favor constantly for *virga* in place of *virgula*, while the other witnesses oscillate without probably meeting its antigraph. The same applies to the variations in the spelling of *addictio*, *additio*, *adictio*, *aditio*, *aditatio* and *additatio*.

Fortunately, some omissions and some additions in the text, which probably were interlinear glosses, allow to reconstruct the first relations of kinship among the codices. Since so far, however, mine are only hypotheses that I have yet to meditate, those presented in this paper are examples of emendations to the *vulgata*'s text, the printed edition of Baldassarre Boncompagni, based on the collation work. In fact, based on my research I have identified variations that offer food for not irrelevant thought. They relate primarily to the *Prologus* of the treatise, if only because it is the most studied section of the work to which I could compare the results of my work with that of the scholars who have preceded me. One interesting point is also offered by the unrest of the transmission of the paragraph concerning the multiplication cross, a significant error that if supported by other similar ones, may in the future allow to clarify the relationship between the codices.

2. *The Prologus of Fibonacci's Liber Abaci and his variations*

In contrast to the greater part of the *Liber Abaci* of Leonardo Fibonacci, which has not had a wide circulation³, his *Prologus*, above all because interesting bi-

² A general summary of the evolution palaeographic of Arabic numerals in Italy is P. Cherubini and A. Pratesi, *L'introduzione della numerazione araba in Italia in età comunale e in Europa. Leonardo Fibonacci e il Libro d'abaco*, in P. Cherubini and A. Pratesi, *Paleografia latina. L'avventura pacifica del mondo occidentale*, Roma 2010, pp. 515-524.

³ Already by this time Niccolò Tartaglia, *General Trattato di numeri, et misure*, Venezia, per Curtio

ographical information can be inferred from it about the author, has enjoyed some considerable circulation in print, while also having been the subject of relatively recent scientific studies. For example, it was published in post note to *Histoire des sciences mathématiques en Italie* of Guglielmo Libri⁴, which merely transcribed the text of the *Prologus* present in the codex Magliabechiano XI 21 of the Biblioteca Nazionale Centrale di Firenze; some years later, in 1857, it was transcribed again, always from a single witness – the manuscript Conv. Soppr. C. 1. 2616 of the same Biblioteca – by Baldassarre Boncompagni, who precisely from this manuscript, prepared the first and still only complete printed edition available up to now of the Fibonaccian *Liber Abaci*⁵. Very recently it has been published in a critical edition that takes all of the manuscript tradition known to date into account. It has also been equipped with an Italian language translation and by a commentary as part of a project which envisions a critical edition of the entire *Liber Abaci*⁶.

As far as the interest of modern scholars, part of the *Prologus* has been the object of a careful study, for example, by Latin scholar Richard Grimm, of the University of California, who focused on the so called “biographical note” of Pisano, that is to say on that part of the *Prologus* from which some of the few certain facts of Fibonacci's life can be gleaned⁷. From the point of view of structure, in fact, the *Prologus* is articulated in two parts: a dedicatory letter to Michael Scotus⁸, followed by a biographical profile which constitutes the principal source of information with which to reconstruct the life of Leonardo Pisano. Proof that the prologue of the *Liber* is in reality divided into two different sections (ded-

Troiano de i Nauò, 1556-1560, had questioned the lack of printed edition of the works of Fibonacci and attributed it to the success of the *Summa de arithmetica geometria proportioni et proportionalità* of Luca Pacioli (Venezia, opificio del prudente homo Paganino de Paganini da Brescia, 1494) who would have «gathered all the flowers»: «Et dicono che la causa di questo [cioè la mancata stampa delle opere di Fibonacci] è processa, perché Frate Luca Patiolo (come che anchora lui medesimo in più luoghi testifica) ne riccolse tutti li fiori, et li interpose nell'opra sua» (cfr. N. Tartaglia, *Quantità, unità, numero. Selezione dal General Trattato di numeri*, ed. by E. Nenci, Milano 2011, p. 13).

⁴ G. Libri, *Histoire des sciences mathématiques en Italie, depuis la renaissance des lettres, jusqu'à la fin du dixseptième siècle*, Paris 1838, p. 289.

⁵ B. Boncompagni, *Scritti di Leonardo Pisano, matematico del secolo decimoterzo*, I-II, Roma 1857-1862 (the first volume contains the edition of the *Liber Abaci*, the second the edition of the *Practica Geometriae*, in addition to those of other minor writers).

⁶ Cfr. G. Germano, C. Carotenuto, *Appendice II*, in E. Burattini, E. Caianiello, C. Carotenuto, G. Germano and L. Sauro, *Per un'edizione critica del Liber Abaci di Leonardo Pisano, detto il Fibonacci*, in *Forme e modi delle lingue e dei testi tecnici antichi*, ed. by R. Grisolia, G. Martino, Napoli 2012, pp. 55-138. In this article I have already referred to the variations of which I will deal more fully explained in this paper, whose main purpose is, in my opinion, spread the most recent research on Fibonacci's *Liber Abaci* also at a wider audience.

⁷ R.E. Grimm, *The autobiography of Leonardo Pisano*, in «Fibonacci Quarterly», 11 (1973), pp. 99-104. I recently heard of a manuscript of the *Liber Abaci*, I do not know yet whether complete or not, just kept in a library California (San Juan de Capistrano, CA, Library of Robert B. Joneyman jr. [Rancho Los Cerritos], General Sci 6). The news is yet to be verified, but it amazes me that Grimm did not mention.

icatory letter and biographical note) can be inferred from the fact that its two parts are treated as independent of one another within the manuscript tradition as it exists to date: in fact, the manuscript I. 72 Sup. of the Biblioteca Ambrosiana di Milano contains only the biographical information (f. 4r); in the just cited Magliabechiano XI 21 manuscript the dedicatory letter is written in smaller point size in the upper right margin of f. 1r, where the biographical note is displayed as well (even if, it seems, it appears to be written by the same hand); in the manuscript L. IV. 20 of the Biblioteca Comunale degli Intronati di Siena, again we find the dedicatory letter (f. 1r), but not the biographical note, while in the other manuscripts (that is, in the above mentioned Conv. Soppr. C. 1. 2616, the manuscript 783 of the Biblioteca Riccardiana di Firenze, the manuscript VIII C 18 of the Biblioteca Nazionale di Napoli and the Palat. Lat. 1343 of the Biblioteca Apostolica Vaticana) both parts appear⁹.

Given the importance of the entire *Prologus* of the *Liber Abaci*, both for its autobiographical content and its introductory function for the treatise itself, I think it opportune to take up again here the study that Grimm made of certain textual variants and extend them to the portion of the text which he omits, that is the dedicatory letter to Michael Scotus. From the collation of the six manuscripts which pass down this portion of the work, some variations emerge which merit further discussion¹⁰.

⁸ Michael Scotus (1175 c.a. - 1235) was a well known personage at the court of Frederick II. Even if his it was uncertain whether his origins were Scottish or Irish for a long period, C.H. Haskins, *Studies in the History of Medieval Sciences*, Cambridge 1927², pp. 273-298, seems to have provided definitive proof of his Scottish origins. According to Haskins Scotus seems to have renounced a profitable office in Ireland because he didn't know Irish. Michael Scotus was born, therefore, around 1190 in Belwearie, in the County of Fife; he studied first at Oxford, then Paris; he visited Toledo around 1217; after 1240 he travelled to Germany, where he met Frederick II, which would have led to his spending some time in Italy at Frederick's court, as well as in other cities, before returning to his country, where he died around 1235. Despite there having arisen a number of legends about this personage (who passed into history as an astronomer and is remembered particularly as having been consulted by Frederick on a range of questions of a naturalistic, theological or generically supernatural character), his most important activities relate to the translation of technical-scientific texts from Arabic. This explains why Fibonacci chose to dedicate his treatise on the abacus to him (cfr. again Germano and Carotenuto, *Appendice II*, in Burattini, *Per un'edizione* cit., pp. 109-112, pp. 128-129).

⁹ It should be noted that, having accounted for the Magliabechiano manuscript XI 21, il Libri, *Histoire* cit., p. 289, is his transcription of the *Prologus* comes after the dedicatory letter to the biographical profile because evidently he didn't know where to locate it. All the other sources that present both parts place the dedicatory letter first and after the biographical profile; since the *Liber Abaci* had two editions one in 1202 and another in 1228 (cfr. what is read in the *incipit* of Riccardiano, Ambrosiano and Vaticano codex: «Incipit liber abaci compositus a Leonardo filiorum Bonaccii pysano in anno MCCII et correctus ab eodem XXVIII»); the same dating also appears in the *incipit* of the Siense manuscript, that contains however some textual variations those previously cited do not, and in the that of the Fond. Princ. II. III. 25 [Magl. XI 22] manuscript of the Biblioteca Nazionale Centrale di Firenze, which is written, however, in the Tuscan vernacular) it is ostensibly to conclude that the dedicatory letter to Michael Scotus was added only with the second draft and is placed at the beginning of the work.

¹⁰ Cfr. the critic edition of *Prologus* in Germano and Carotenuto, *Appendice II*, in Burattini, *Per un'edizione* cit., pp. 122-124.

3. *A textual variant: magis ad theoreticam spectat quam ad practicam*

The first interesting case appears in the dedication to Michael Scotus, specifically in that portion of the text of the *Prologus* that, as we have seen above, wasn't subjected to any philological analysis by Grimm: in a passage in which Fibonacci tries to explain briefly the purpose of his treatise, the tradition diverges at a fairly crucial point in order to define precisely the type of work that Pisano claims to have done. After having asserted that in his *Liber* he would expound the «entire doctrine of numbers according to the method of the Indians»¹¹, and stated that in the treatise geometric examples would be utilised, since «arithmetic and geometry are linked and bear each other out»¹², Leonardo adds, according to the text set forth in the recent critical edition, that his book is more concerned with practice than with theory, since to best assimilate the theoretical concepts one must practice diligently over a long period. Fibonacci, essentially, maintains that his treatise is more practical than theoretical: therefore, it is precisely this claim I wish to analyse, since in most of the manuscripts we find exactly the lesson «magis quam ad theoreticam spectat ad praticam»¹³. In the manuscript Napoletano VIII C 18 we find a different formulation, but of identical meaning, specifically «ad practicam magis quam ad theoreticam spectat», while in only the the single Fiorentino Conv. Soppr. C. 1. 2616 he reverses this conception with his lesson «magis ad theoreticam spectat quam ad practicam», misrepresenting, in my opinion, that which Leonardo wanted to express¹⁴. Since this codex represents the source used by Boncompagni for his printed edition and is therefore that upon which the text of the Fibonacci treatise today taken to be authoritative is based, all the studies conducted on this tract refer to the alleged claim of Fibonacci that the *Liber Abaci* «is concerned more with theory than practice». This has caused more than a little confusion about the objective of the treatise.

What precisely is the nature of the *Liber Abaci*, has in fact been a question often debated by scholars, a debate probably fueled by the apparent incongruence between the declaration of the *vulgata*, according to which it is more a theoretical than a practical manual, and the great attention that the author instead gives to practical questions. In any case, close consideration should be given to the form of the statement by Leonardo about the essentially practical nature of his treatise: he does not write that his is a practical manual *tout court*, but only that it «looks to», «refers to», and «concerns» the practical, namely that

¹¹ «Plena numerorum doctrinam edidi iuxta modum Indorum»: cfr. Germano e Carotenuto, *Appendice II*, in Burattini, *Per un'edizione cit.*, p. 122.

¹² «Et quia arismetica et geometrica scientia sunt connexe et suffragatorie sibi ad invicem»: cfr. Germano and Carotenuto, *Appendice II*, in Burattini, *Per un'edizione cit.*, p. 122.

¹³ This expression is seen in the Riccardian 783, Siense L. IV. 20, Vatican Palat. Lat. 1343 e Magliabechiano XI 21 manuscripts.

¹⁴ For these variants cfr. Germano, Carotenuto, *Appendice II*, in Burattini, *Per un'edizione cit.*, p. 122, in apparatus.

in this it accomplishes its effective task. In fact, Pisano does not shy away from dealing with mathematical questions of a theoretical type, even if he never loses sight of their practical application¹⁵. Concerned with this issue, for example the historian of medieval Mediterranean Marco Tangheroni, noting that the *Liber Abaci* devotes much space to problems of arithmetic involved in the practical work of merchants, even if «it cannot be considered a practical manual for use by merchants: the *Liber* does not provide formulas for immediate application, but rather problems of, and solutions for problems through, mathematical procedures»¹⁶. Certainly a manual on mathematics cannot avoid appearing to be theoretical, even if the space Fibonacci devotes to practical questions is significant: the middle chapters (VIII-XII) are entirely dedicated to them and, as when he explains algebraic operations, Leonardo uses diverse examples and, before proceeding with a new topic, recommends extensive practice to order to both assimilate what has been taught, and to understand the concepts to follow. Another scholar, Antonio Antoni points out that for the bulk of information relative to the uses of different markets, covered in chapters VIII to XII, we can compare this part of the *Liber Abaci* to a “Trading Guide” *ante litteram* (the oldest “Trading Guide” that has come down to us was compiled by Francesco of Balduccio Pegolotti, estate manager of the Bardi in 1340)¹⁷. Ultimately, the *Liber Abaci* seems to be a hybrid composition: Leonardo is the first to introduce practical problems, but in his manual there is still much of the theory that will disappear from all the successive treatises on the abacus, which are completely practical often consisting of a sort of list of directions for the use of merchants¹⁸. It is not by chance, as Raffaella Franci¹⁹ notes, «The output of mathematics of the High Middle Ages developed in the wake of the *Liber Abaci*, was ignored, even scorned, by the official culture». If therefore, «Leonardo effected a commercial revolution, seeking in the study of numbers tools of irreplaceable

¹⁵ On the other hand that of Fibonacci could be a declaration of false modesty, which was a *topos* of medieval literature (cfr. E.R. Curtius, *Letteratura europea e Medio Evo latino*, ed. it. by R. Antonelli, Firenze 1992, pp. 97-100). Fibonacci would have been able to use this expedient to avoid criticism by those who compared his work to that of Boethius or of the same Euclid.

¹⁶ «Esso non può essere considerato un manuale pratico ad uso dei mercanti: il *Liber* non offre formule di immediata applicazione, ma problemi e soluzioni di problemi attraverso procedimenti matematici»: M. Tangheroni, *Fibonacci, Pisa e il Mediterraneo*, in *Leonardo Fibonacci. Il tempo, le opere, l'eredità scientifica*, ed. by M. Morelli and M. Tangheroni, Pisa 1994, p. 25.

¹⁷ A. Antoni, *Leonardo Pisano detto il Fibonacci e lo sviluppo della contabilità mercantile del '200*, in *Il tempo cit.*, p. 46.

¹⁸ U. Tucci, *Manuali d'aritmetica e mentalità mercantile tra Medioevo e Rinascimento*, in *Il tempo cit.*, p. 55, writes in reference to the abacus treatise of G. Sfortunati, *Nuovo Lume, libro di aritmetica*, Venezia, Bernardino Bindoni, 1545: «Sfortunati, che pure conosce la concezione che ne hanno Euclide, Boezio e Il Sacrobosco, tralascia la spiegazione di cosa sia il numero risparmiandola al suo pubblico *per essere cosa al mercante laboriosa e difficile*».

¹⁹ R. Franci and L.T. Rigatelli, *Introduzione all'aritmetica mercantile del Medioevo e del Rinascimento, realizzata attraverso un'antologia degli scritti di Dionigi Gori (sec. XVI)*, Urbino-Siena 1982, p. 22.

value for the market»²⁰, the impression one has in reading the later books about the abacus is that arithmetic served only as preparation for the defining and solving of problems encountered in mercantile activity. Even later, in the treatise tradition of the abacus the exercises lose their direct connection to the actual circumstances of mercantile activity and shift instead to advancing the subject of calculation for the purposes of entertainment. In this way was defined

a separation between a commercial arithmetic and one generically defined as *practical* or *elementary*, as opposed to a speculative kind, accepting that this label can be applied to the works of Boethius embraced by medieval ecclesiastical schools and cited in the abacus treatises in order to enoble the text with references to tradition²¹.

On the other hand it would not be fruitless to reproduce here an excerpt of the *General Trattato di Numeri e Misure* of Niccolò Tartaglia that, in 1556, systematically explains the difference between mathematical theory and practice²²:

Le specie della Arithmetica sono due, cioè theorica et pratica. La theorica considera le cause, le qualità, le quantità, et le proportion de numeri con una speculation di mente, et il suo fine non è altro che la verità, et di questa abbondantemente ne tratta il nostro precettore Euclide Megarese nel suo settimo, ottavo et nono libro, delli quali al suo luoco et tempo in pratica ne parleremo. La pratica poi considera solamente l'attione, over calculatione, et il fin suo non è altro che il compimento di tal attione, over calculatione; et di questa pratica è lo intento nostro di voler abodantemente trattare; incominciando prima dalle prime attoni, pratiche, et regole generali, et particolari, pertinenti a tutta l'arte negociaria, over mercantile²³.

In conclusion, this case appears particularly instructive, since it enables us to understand how a trivial error in a manuscript, passed down in a printing of an authoritative version of the work, has influenced the views of scholars and created misunderstandings regarding the inconsistency between what seems to be the author's opinion as expressed in the *Prologus*, versus what is effectively the reality of the work. Restoring what seems the correct and authentic lesson solves many of the problems that were outlined in the discussion concerning the nature of the Fibonacci treatise.

²⁰ T. Fanfani, *Brevi note in margine ad un convegno*, in *Il tempo* cit., p. 42.

²¹ Tucci, *Manuali d'aritmetica* cit., p. 51f.

²² Tartaglia, *Quantità* cit., p. 34.

²³ «The typologies of Arithmetic are two, theoretical and practical. The theoretical considers the causes, the qualities, the quantities, and the proportions of numbers by means of speculative investigations, and its goal is none other than the truth; and of this our preceptor Euclid of Megara treats in depth in his seventh, eighth and ninth books, about which we will speak in the proper place. The practical on the other hand considers only action, or rather calculation and its purpose is the performance of this action, or rather calculation. Our intent is to deal with the practical at great length, beginning first with prime actions, practice, and rules, both general and particular, pertaining to the whole of the financial, or rather commercial, art».

4. A textual and exegetical problem: The Arcus Pictagore

The other two cases I intend to discuss have been study subjects which Grimm carried out on the biographical note of the *Prologus*²⁴. It has to do with two textual variants that appear in a short series of sentences in the part of the *Prologus* in which Fibonacci reviews the studies he conducted during his youth, when, accompanying his father, a state official in Bejaïa, he was able to enter into contact with the best teachers of mathematics of his time and was instructed for the first time in the Indo-Arabic mathematical methods; he continued to study these in more depth in the course of his peregrinations to the principal cultural, and commercial, centres of the Mediterranean. In this context, therefore, to start with the most meaningful case, Leonardo extolled what he called «the Indian method» by contrasting it with the «algorismus» and the «arcus Pictagore».

Precisely what the «arcus Pictagore» was wasn't immediately clear to scholars; so too was the exegesis of this expression complicated by the fact that the tradition on this subject is not in agreement. The «arcus» lesson only appears in the Florentin Conv. Soppr. C. 1. 2616 and in the Riccardian 783 manuscripts, but is absent instead in the Vatican Palat. Lat. 1343, the Napoletano VIII C 18, in the Ambrosiana I. 72 Sup. and the Magliabechiano XI. 21 manuscripts. Grimm, however, maintains that the lesson appears only in the Conv. Soppr. C. 1. 2616 manuscript and is doubtful in the Riccardiano 783, where he states that in its place one reads «artem». Having, however, personally collated the manuscripts, I maintain that no doubts exists that the «arcus» lesson is present in this manuscript as well. Grimm, notwithstanding this, and having succeeded in discovering the meaning of the expression «arcus Pictagore», holds that the «arcus» present in Conv. Soppr. C. 1. 2616, is a transcription error in place of «artem», basing this conclusion on the simplistic premise that in medieval script the *t* and the *c* can be easily confused. He arrives at this conclusion after observing that in the passage reference is made to the “art” of the Indi²⁵ and the “art” of Euclid²⁶: according to him, therefore, a reference to the “art” of Pythagoras is probable. Furthermore, Grimm, always based on questionable paleographic analysis, attempts to trace the origin of this omission, which according to him would be «artem», to the majority of the original manuscripts containing the lesson. He asserts that the scribes, because the terms «artem» and «atque» in the writing of the 13th and 14th centuries were abbreviated in a similar manner²⁷, would have read «atque atque e», thinking to have identified a duplication error, and consequently would have omitted one of the two words²⁸.

²⁴ Grimm, *The autobiography* cit., pp. 99-104.

²⁵ «Ubi ex mirabili magisterio in arte per novem figuras Indorum introductus» (cfr. Germano, Carotenu-
to, *Appendice II*, in Burattini, *Per un'edizione* cit., p. 124).

²⁶ «Quedam etiam ex subtilitatibus Euclidis geometrice artis apponens» (cfr. Germano, Carotenu-
to, *Appendice II*, in Burattini, *Per un'edizione* cit., p. 124).

²⁷ This assertion of Grimm does not seem to be borne out by the abbreviations catalogued in A. Cap-
pelli, *Dizionario di abbreviature latine ed italiane*, Milano 1979, pp. 24, 26.

²⁸ Grimm, *The autobiography* cit., p. 103.

In reality, Grimm, having had principally humanistic training, erred in his evaluation of these variants above all because he ignored, as I have just demonstrated, what the «arcus Pictagore» consists of. Actually, the expression is not immediately understood: probably the scribes did not know the meaning which would explain the absence of the «arcus» lesson in the majority of the manuscript sources transmitted to us. Only research on the history of the medieval mathematics has allowed me to identify the «arcus Pictagore» as a calculation tool, the abacus developed by Gerbert of Aurillac²⁹, and to understand, therefore, that Fibonacci refers here, for metonymy, to the same mathematics taught by the teachers of the abacus³⁰. The denomination «arcus Pictagore» is traced back to the legend, also recalled by the British scholar E.C. Bayley, according to which the introduction to the world of the *abacus*, understood as a tablet covered with sand used to perform calculations, is actually owed to Pythagoras³¹. On the other hand the Indians were already using a tablet covered with sand to trace geometric figures and to perform calculations. Bayley, in fact, records that the most common etymology of “abacus” links the name of the tool with an ancient Semetic word meaning “fine sand”: it follows from this that the instrument must have had from the beginning the form of a tablet with a raised border to hold the sand covering it³². Around the 11th century Gerbert updated this model which introduced numeric symbols in place of pebbles. Gerbert’s tablet – almost as if were an abacus understood as a bullet holder – was divided into thirty vertical columns (three of which represented decimals) formed into groups of three by semi-circular arcs drawn above them; every column was surmounted by a small arc in which was written *I* for single digits, *X* for tens and *C* for hundreds: from this derives the expression «Arcus Pictagore» or simply «Arcus»³³. At the time of Fibonacci, therefore, in Europe the calculations were made using the Gerbert’s *abacus*, while the Arabs in North Africa, he was started on a new way forward, faster, in which the operations were summarized by tracing patterns on a tablet: these are the patterns that Leonardo exemplifies

²⁹ C. Burnett, *The semantics of indian numerals in arabic, greek and latin*, in «Journal of Indian Philosophy», 34 (2006), p. 19.

³⁰ Cfr. J.W. Durham, *The introduction of Arabic numerals in European accounting*, in «The Accounting Historians Journal», 19 (1992), 2, p. 28, maintains that from the beginning of the 11th century, the term “abacus” is also used by extension to denote the subject of calculation in general.

³¹ Cfr. E.C. Bayley, *On the genealogy of Modern Numeral*, in «Journal of the Royal Asiatic Society of Great Britain and Ireland», New Series, 15 (1883), 2, pp. 1-72. Bayley (*On the genealogy* cit., p. 8) recalls that the popular belief of the Greeks that the abacus was introduced in Greece by Pythagoras, and that Giamblico – even if he writes at a relatively late date – states that it was using the abacus that Pythagoras would have taught both arithmetic and geometry.

³² Bayley, *On the genealogy* cit., p. 9.

³³ D.E. Smith and L.C. Karpinski, *The Hindu-Arabic Numerals*, Mineola 2004, p. 122; M. Bramanti, *L'abaco di Gerberto e l'apprendimento della scrittura posizionale dei numeri*, in «Emmeciquadro», 34 (2008), p. 67f. On the use of the term «arcus» to indicate the columns of the abacus cfr. G.R. Evans, *From Abacus to Algorithm: Theory and Practice in Medieval Arithmetic*, in «British Journal for the History of Science», 10 (1977), 2, p. 118.

constantly on the sidelines of his treatise, just a testimony to the fact that it was a new procedure.

In this regard, read the following passage taken from chapter II, devoted to multiplication. Fibonacci has just explained the new procedure to perform the calculations of multiplication and now he exemplifies numerically, by the end, reference to the figure in the margin.

Si quesierit multiplicationem de 12 in 12, scribatur 12 bis in tabula dealbata, in qua lettere leviter deleantur, sicuti in hac margine scriptum cernitur, primus gradus subterioris numeri sub primo superioris, hoc est figura binarii sub figura binarii, et secundus gradus subterioris sub secundo superioris, scilicet figura unitatis sub figura unitatis, et multiplicet binarium per binarium, erunt 4, que ponat super utrumque binarium ut in prima descriptione posita sunt. Iterum multiplicentur superiora 2 per 1 qui est in secundo gradu inferioris numeri, erunt 2, que servantur in manu. Et multiplicentur iterum 2 subterioris numeri per 1 superioris, erunt 2 que addat cum duobus superius servatis, erunt 4, que ponat super unitatem utramque, facient ipsa 4 secundum gradum, post priora posita 4 que fecerant primum gradum, ut in secunda descriptione describitur; et adhuc multiplicetur 1 de superiori numero per unum de subteriori, faciet 1. Quod 1 scribatur in tertio gradu, scilicet post 44 descripta, ut in tertia et ultima descriptione ostenditur. Et in tot ascendit multiplicatio de 12 in se ipsa: scilicet 144³⁴.

5. *The pitfall of textual variations that are unsuspected: tam in place of causa*

On the other hand, similar paleographic pretexts, such as the confusion between *t* and *c*, are used by Grimm, but in this case properly. So too in the analysis of another textual variant that appears just before the reference to the «arcus Pictagore», where Pisano, listing the countries where he has acquired his mathematical knowledge, specifies that in those places «negotiationis causa postea peragravi»³⁵. In this part of the text, the lesson «causa» appears in all the manuscripts except Conv. Sopp. C. 1. 2616, which substitutes for it the lesson «tam».

³⁴ B. Boncompagni, *Scritti di Leonardo Pisano*, Roma 1854, pp. 7-8. The text is by me provisionally established on the basis of the collation of a part of the tradition. «If one will wish to find the multiplication of 12 by 12, then 12 is written in the chalk table in which the letters are easily deleted, as is shown written in this margin; the first place in the lower number is below the first place in the upper, that is the figure two below the figure two, and the second place in the lower below the second in the upper, namely the figure one below the figure one, and the two is multiplied by the two; there will be 4 that is put above both of the twos, as is placed in the first illustration. Again the upper 2 is multiplied by the one which is in the second place of the lower number; there will be 2 which is kept in hand, and again the 2 in the lower number is multiplied by the 1 in the upper; there will be 2 which one adds with the above held two; there will be 4 that is put over each unit which makes the 4 in the second place after the prior put figure 4 making the first place, as is written in the second illustration, and also the 1 in the upper number is multiplied by the one in the lower making 1; this is written in the third place, namely after the written 44, as is shown in the third and last illustration. And in this total results the multiplication of 12 by itself, namely 144» (L.E. Sigler, *Fibonacci's Liber Abaci. Leonardo Pisano's Book of Calculation*, New York 2002, p. 24).

³⁵ Germano, Carotenuto, *Appendice II*, in Burattini, *Per un'edizione cit.*, p. 124.

From the moment that, as already stated, Boncompagni used only this codex for his printed edition, the error of one manuscript was attested to in the “bible” of the Fibonacci treatise.

As is noted by Grimm, who correctly restored the lesson *causa* in place of *tam*, this small error was rather insidious as it permitted an easier translation: «in which places of commerce I afterward long travelled»; even though he noticed that *tam postea* seemed an inferior Latin with respect to the classical variety, which would be, according to him, «tanto postea»³⁶. The collation of manuscripts, however, allowed the question to be resolved quickly, showing that the «tam» of the above mentioned Florentine codex is simply a corruption – immediately evident from the point of view of paleography – of *causa* due to the fact that the two terms have a similar abbreviation³⁷. Once the original lesson is restored, the translation of the passage being examined – having become «places where afterward I travelled for the purpose of commerce»³⁸ – has added a further biographical detail of Leonardo's life: that the primary purpose of his travels was commercial activity, even if Pisano then took advantage of the opportunity to meet the most important mathematicians of the era to enhance his knowledge of the subject. Now, discuss what in reality is nothing but a singular lection easily amendable according to the consensus of the other codes, it may seem overwhelming. If, however, I took care not only to return to a point already covered by Grimm and myself all shared, but especially to be noted that this example demonstrates once more that Boncompagni's text, even where it does not seriously distort the original, should be revised, so clear in this case, where he ascribes to Pisano a barbarism solely because of a corruption of the manuscript tradition: a new edition, a critical one this time, of the entire *Liber Abaci* could without question reveal many surprises as well as clarify, perhaps, a good number of the disputed passages of Fibonacci's work.

6. Behind the Prologus: an important example of transposition, “cross multiplication”

Another error of the vulgata's text, that the simple collation of manuscripts has easily allowed me to amend, is the transposition of a paragraph of chapter II that the source of Boncompagni puts, however, after the first paragraph of chapter III. The chapter II of *Liber Abaci* is dedicated to multiplication, while chapter III is devoted to additions.

Reading the codex *F*, it is confusing the fact that Fibonacci, after having started the third chapter dealing with the addition, going off track, returns to the multiplication describing a different method to do it: what is defined by the schol-

³⁶ Grimm, *The autobiography* cit., p. 102.

³⁷ The abbreviations of «tam» and «causa» are respectively *ta* and *ca* and their prime letters *t* and *c* can be confused: cfr. Cappelli, *Abbreviazioni* cit., pp. 41, 371.

³⁸ Germano, Carotenuto, *Appendice II*, in Burattini, *Per un'edizione* cit., p. 125.

ars «the method of cross multiplication». In fact at the beginning of chapter II Fibonacci writes clearly that this chapter will be divided into eight sections³⁹, but there is no trace of the eighth section in the source of Boncompagni and then in the vulgata's text.

Est enim alius modus multiplicandi valde laudabilis, maxime in multiplicandis magnis numeris, quem ostendam in multiplicatione de 567 in 4321. Constituatur quadrilaterum in forma scacherii, habens puncta 5 in longitudine, scilicet unum plus numero figurarum maioris numeri, et habeat in latitudine puncta 3, sicuti sunt tres figure in minori numero. Et ponatur maior numerus super quadrilaterum supradictum, et minor ponatur ante ipsum, ut hic cernitur [...]⁴⁰.

This paragraph, which constitutes, in fact, the eighth part of chapter II presents the following codicological situation:

- appears at the close of chapter II under the title *pars octava* in the codices *S* and *V*;
- appears at the close of chapter II, but under the title *pars septima* in *R* codices;
- appears almost in the same position, but under the title *Incipit capitulum Tertium* in *NA F*;
- appears in different position only in *F*, which puts it after the first paragraph of chapter III.

Therefore, although the title *pars octava* appears only in the codes *S* and *V*, it is certainly to be accepted in the critical text, after it has been definitely re-established the position of the paragraph in question indicated by *consensus codicum*, but also deductible by the logic itself of the text.

7. Conclusions

The variations discussed in this paper do not allow to draw firm conclusions on the family relationship between the codices. Nevertheless, based on the collation work that so far I have brought to an end and that includes the first six chapters, I would venture in a guess that, although I think it should be further explored and supported by subsequent investigations, can provide interesting points for reflexion by other scholars.

³⁹ «Capitulum secundum de multiplicationibus integrorum numerorum in octo partes dividimus, ut differentie atque proprietates earum melius intelligantur» (Boncompagni, *Scritti* cit., p. 7).

⁴⁰ Boncompagni, *Scritti* cit., p. 19. «There is indeed another way of multiplication greatly praised, best for multiplying large numbers, which I shall show in the multiplication of 567 by 4321. A rectangle is constructed in the form of a chessboard having 5 point in length, namely one more than the number of figures of the greater number, and having 3 point in width, as there are three figures in the smaller number, and the greater number is put over the abovesaid rectangle, and the smaller is put before it, and this is displayed» (L.E. Sigler, *Fibonacci's Liber Abaci* cit., p.39). For brevity of the summons I did'nt put the full text of the paragraph that continues with a detailed description of this operation.

In particular, it seems to me we can identify kinship closer union among $AVNF_1^{41}$, while other codices belong to another branch of the tradition. In fact, the assumption on which I feel more certain is the very close relationship observed between the N and F_1 , so that practically all variations of F_1 also appear in N , and where they do not appear they can be justified as a correction of the copyist. I will only mention a single example from chapter VII of *Liber*, which is dedicated to addition and subtraction of fractional numbers:

Si vero de $\frac{1}{4}$ $\frac{1}{8}$ de $\frac{1}{4}$ $\frac{1}{8}$ extrahere volueris⁴².
[vero $AFRSV$, volueris F_1N ; volueris $AFRSV$, om. F_1N]

Yet, it is a difference of variations between N and F_1 , and the palaeography motivation on which I think it is based, to make me lean towards a derivation, not necessarily direct, of N from F_1 . To this aim note, always in chapter VII, the following passage:

Si volueris addere $\frac{1}{3}$ 12 cum $\frac{3}{4}$ 126, describe numeros ut hic ostenditur, et multiplica 12 per suam virgulam.
[multiplica 12 FF_1VRSA , multiplica ea 12 N]

The expression «multiplica 12» is passed in the whole tradition except in N where it is stated clearly «multiplica ea 12». This singular lection might be induced in N by the copyist misreading of F_1 where the writing of «multiplica» is ambiguous: in particular, «ca» may have been confused with «ea» from N or its lost antigraph.

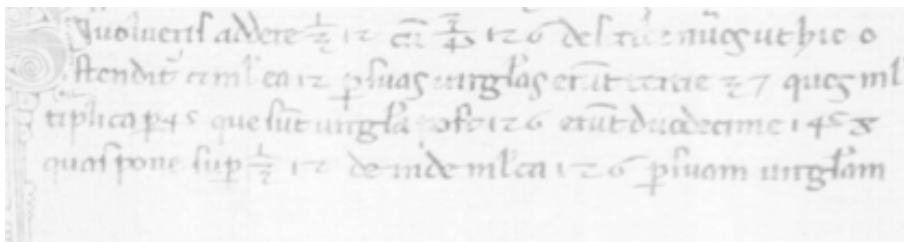


Fig.1. Firenze, Biblioteca Nazionale Centrale, ms. Magl. XI. 21, f. 50v.

⁴¹ It seems misleading to the fact that the section of the cross multiplication to appear under the title *pars octava* own in S and V that otherwise seem to belong to two distinct branches of the tradition. But if my hypothesis should not be revised in the light of further developments in research, the coincidence of class S and V may be due to corrections autonomous conjectural in one or more of the surviving witnesses. Once again, to resume the discussion of the introductory paragraph, you will understand how difficult it is to establish the actual relationships among the witnesses in a work of a technical nature such as the *Liber Abaci*.

⁴² Boncompagni, *Scritti* cit., p. 66.

In conclusion, considering what already expressed in the introduction about the difficulty of clarifying the actual relationships among the witnesses of a work of technical nature, I want to point out, once again, that my assumptions could be reversed from future results of the research.

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