Giuseppe Germano

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1. *The importance of the* Liber Abaci *in the history of western culture*

The *Liber Abaci* of Leonardo Pisano, called the Fibonacci[1], plays, as is already well-known, a founding role in the history of mathematics and can be considered without any doubt a seminal work in the history of medieval Europe. In fact, even if purely and simply from the point of view of mathematical knowledge the treatise written by Fibonacci does not appear to surpass the achievements already attained in this field by Arab scholars in the 10th century[2], it was not only able to establish a fruitful connection between two cultures, christian and arab, which up to that point were profoundly discordant even with respect to the level of knowledge and methods of mathematical calculation, but also with its enormous size, its systemic clarity and coherence, it constituted a wondrous compendium of everything that had been accomplished in its field from antiquity up to that time. Thus, in its own era, it contributed incisively to the development of mathematical thought and the broadening of cultural horizons, building a bridge between the rig-

[1] For a up-to-date synthesis of the biography and the work of Fibonacci and, in particular, on the composition, the content and the cultural significance of his *Liber Abaci*, E. Caianiello, *La vita e l'opera di Leonardo Pisano*, in E. Burattini, E. Caianiello, C. Carotenuto, G. Germano and L. Sauro, *Per un'edizione critica del* Liber Abaci *di Leonardo Pisano, detto il Fibonacci*, in *Forme e modi delle lingue e dei testi tecnici antichi*, ed. by R. Grisolia and G. Matino, Napoli 2012, pp. 59-85, for the *Liber Abaci*, pp. 65-72.

[2] Such is the opinion expressed in particular by R. Rashed, *Fibonacci et les mathématiques arabes*, in «Micrologus», 2 (1994), pp. 145-160; R. Rashed, *Fibonacci et le prolongement latin des mathématiques arabes*, in «Bollettino di storia delle scienze matematiche», 23 (2003), 2, pp. 55-73.

orous Euclidean methods of proof and the pragmatic mentality of the commercial world to which Leonardo himself belonged[3].

Among the most noteworthy achievements of the Fibonacci's *Liber Abaci* is that of unifying diverse traditions, such as that of the masters of calculation with those of algebra, establishing with the rigor of a scientific method and in writing a large number of algorithms which prior to that time had been calculated only empirically, by observation and mental reckoning alone. Other achievements were the dissemination of the use of fractions in the West, which up to then in the academic world had always been viewed with suspicion; progress in the use of new notions of numbers (such as the concepts of negative numbers or zero); contribution to the development of linear systems and the resolution of second degree equations, laying the groundwork for the subsequent renewal of mathematics in the western world and for the splendid progress in algebra which would occur in Italy in the middle and late Renaissance[4]. Moreover, as a commercial manual the *Liber Abaci* holds a position of unquestioned supremacy both for its depth of knowledge about coins and trade practices of its time, and for its originality of presentation of mathematical procedures such as the amortization of a burdensome loan or the discounting of an amount: thus, because its influence endured for almost three centuries after its publication it is to be considered a true classic of commercial mathematics[5].

2. *The manuscript tradition of the* Liber Abaci *to date*

The text of the *Liber Abaci* is still for the most part found in the papers or parchments of the manuscript codices, which comprise only a meagre handful of nineteen units scattered mostly among various French and Italian libraries. These manuscripts are known on the basis of studies made on this text over the last two centuries, which are largely of a historical and mathematical nature; however their number is likely to grow, given that up to now no specific and systematic investigations on the manuscript tradition of the *Liber Abaci* have been carried out and at the moment are still going by the working group headed by the writer[6].

[3] E. Giusti, *Matematica e commercio nel* Liber Abaci, in *Un ponte sul Mediterraneo: Leonardo Pisano, la scienza araba e la rinascita della matematica in Occidente*, ed. by E. Giusti and R. Petti, Firenze 2002, pp. 59-120, specially pp. 59-60 (now accessible also online at the url <http://php.math.unifi.it/archimede/archimede/fibonacci/catalogo/giusti.php>).

[4] K. Vogel, *Fibonacci, Leonardo or Leonardo of Pisa*, in *Dictionary of Scientific Biographies*, New York-Oxford 1970-1980, IV, pp. 604-613 (now also available online: Vogel, *Fibonacci, Leonardo, or Leonardo of Pisa*, in *Complete Dictionary of Scientific Biography*, 2008, *Encyclopedia.com*, <http://www.encyclopedia.com/doc/1G2-2830901418.html>).

[5] F. Melis, *Storia della ragioneria. Contributo alla conoscenza e interpretazione delle fonti più significative della storia economica*, Bologna 1950, *passim*; F. Melis, *Industria e commercio nella Toscana Medievale*, ed. by B. Dini and M. Tangheroni, Firenze 1989, *passim*.

[6] E. Caianiello, *La tradizione manoscritta del* Liber Abaci *di Leonardo Pisano*, in Burattini, *Per un'edizione* cit., pp. 85-88, from which I have taken the list of manuscripts and particulars con-

Such manuscript witnesses can be divided into two distinct groups. The first include those transmitting the Fibonaccian treatise in its entirety; in the second can be counted those transmitting the text in an incomplete manner or even only parts of it.

The witnesses of the first group are the following:

Berlin, Staatsbibliothek, Preußischer Kulturbesitz, ms. Lat. Fol. 418, ff. 1-805, paper, 17th century[7].

Firenze, Biblioteca Nazionale Centrale, ms. Conv. Soppr. C. 1. 2616, ff. 1-214, parchment, early 14th century.

Firenze, Biblioteca Nazionale Centrale, ms. Magl. XI. 21, ff. 1-285, parchment, 14th century.

Firenze, Biblioteca Riccardiana, ms. 783, ff. 1-346, paper, 15th century.

Milano, Biblioteca Ambrosiana, ms. I. 72 Sup., ff. 1-124, parchment, 13th century (missing the dedicatory *Epistula* to Michael Scotus).

Napoli, Biblioteca Nazionale, ms. VIII. C. 18, ff. 1-285, paper, 17th century.

The manuscript witnesses of the second group can be further subdivided, since two of them present the text in almost complete form, while the others contain only single chapters or parts of them.

The manuscripts that display the text of the *Liber Abaci* almost in their entirety are the following:

Firenze, Biblioteca Nazionale Centrale, ms. Fond. Prin. II. III. 25 [Magl. XI. 22], ff. 1-175, paper, 15th or 16th century (missing the dedicatory *Epistula* to Michael Scotus and a good part of chapter XV).

Siena, Biblioteca Comunale degli Intronati, ms. L. IV. 20, ff. 1-224, parchment, ending 13th - early 14th century (missing the autobiographical *Prologus* and a part of chapter XV).

Città del Vaticano, Biblioteca Apostolica Vaticana, ms. Palat. Lat. 1343, ff. 1-174, parchment, ending 13th - early 14th century (missing the chapter X).

The manuscripts that transmit instead only portions of the Fibonaccian treatise are indicated here:

Firenze, Biblioteca Medicea Laurenziana, ms. Gadd. 36, ff. 1-168, paper, 14th century (transmits only the last four chapters of the treatise, those from XII to XV, the last of which is also incomplete as it terminates in the middle of the third part).

Firenze, Biblioteca Nazionale Centrale, ms. Magl. XI. 38, ff. 120*r*-231*v*, paper, 16th century (transmits only the last two chapters of the treatise, the XIV and the XV).

cerning their composition and to which I refer for further details and for the bibliography. The Berlin and Californian manuscripts were not part, however, of this list, which consisted of seventeen units, and must, therefore, be added to it: they have been identified by Concetta Carotenuto in the course of a research, that is aimed at the preparation of a doctoral dissertation on Fibonacci's *Liber Abaci* and that she is carrying under my personal mentoring to the award of PhD in *Filologia classica, cristiana e medioevale-umanistica, greca e latina* at the University of Naples Federico II.

[7] I don't agree with the repertoire *Mirabile* (available *on-line* at the url <http://www.mirabileweb.it/>) regarding the dating of the ms., that is there established, in my opinion wrongly, to the 15th century.

Firenze, Biblioteca Riccardiana, ms. 2252, ff. 72*r*-142*r*, paper, 14th century (transmits the greater part of chapters XII and XIII of the treatise; the copy begins in the Tuscan vernacular, but then continues, from f. 107*v*, in Latin).
Paris, Bibliothèque Mazarine, ms. 1256, ff. 33*r*-85*v*, paper, 14th century (transmits only chapters XIV and XV).
Paris, Bibliothèque Nationale, ms. Lat. 7225 A, ff. 81*r*-220*r*, paper, 16th century (transmits only chapters XIV and XV).
Paris, Bibliothèque Nationale, ms. Lat. 7367, ff. 1-168, paper, 15th century (transmits only chapters XIV e XV of the treatise, after which follows a description in the Tuscan vernacular of the *numeri rupti*, that is of fractions).
Città del Vaticano, Biblioteca Apostolica Vaticana, ms. Urb. Lat. 291, ff. 1*r*-33*v*, 121*r*-132*v*, paper, 14th century (transmits a part of chapters XIV and XV in the Tuscan vernacular).
Città del Vaticano, Biblioteca Apostolica Vaticana, ms. Vat. Lat. 4606, ff. 52*r*-71*v*, 77*r*-107*v*, paper, 14th century (transmits parts of chapters XIV and XV).

A separate position must be attributed, finally, to the two manuscript witnesses which have remained outside this classification: the first is a treatise in the Tuscan vernacular divided into 16 chapters with a re-elaboration of Fibonacci's *Liber Abaci* enriched with insertions by Benedetto of Florence from other arithmetical texts:

Siena, Biblioteca Comunale degli Intronati, ms. L. IV. 21, ff. 1-208, paper, 15th century;

of the second, as stored in an United States private library, it has been possible up to now to obtain only a few information and I can't say, therefore, if it contains the Fibonacci's treatise in whole or only in part:

San Juan de Capistrano, CA, Library of Robert B. Joneyman jr. (Rancho Los Cerritos), ms. Gen. Sci. 6, ff. 104-221, paper, 14th-15th century[8].

These are, as is clear from a cursory review of the few particulars presented in the list above, witnesses that appear very different from the standpoint of textual breadth and which do not allow for a facile orientation to the modality of the tradition of the treatise; except, however, in the case of a bit of fortune encountered by the concluding chapters of the *Liber Abaci*, which deal with the erratic questions and their numerous problems within indeterminate systems, as well as those theoretical questions constituting the first valid treatment of algebra expressed in the Latin language. Moreover such witnesses, whose value cannot be established *a priori*, since no one has ever attempted as yet to study the question of their reciprocal relationships and their possible connections from the philological-textual point of view, often pose somewhat delicate problems which require a range of specialized skills: this is the case, for example, of the manuscripts which present either a text in the Tuscan vernacular variously in-

[8] From the repertoire *Mirabile* (available *on-line* at the url <http://www.mirabileweb.it/>) we can derive not only the data that I have reported, but also the size of the ms. (mm. 220 x 160): it consists in its entirety of 222 ff. and contains to the ff. 1-64, as it seems, the *Institutio Arithmetica* of Boethius; but I could not say at the time what contain its ff. 65-103.

tegrated and/or cut with respect to the original, or one characterized by the presence of both Latin and the Tuscan vernacular, and that therefore, so configured, can be seen as evidence that the tradition of the Fibonacci's treatise is a living one[9].

Of these manuscript witnesses, within the working group headed by the writer, have been partially investigated at the time, through a systematic collation[10], only the ones containing the work in its complete or nearly complete form: in this phase of the work, which seems slow and presents many difficulties, it doesn't seem scientifically correct to go too far in a premature evaluation on their possible relationships and in an albeit temporary building of a *stemma codicum*. That may be risky and be subsequently disproved in the light of other possible acquisitions in the progress of the work, as is the experience of all those who perform research in the field of textual criticism. Although, as it will be said later, some reasonable working hypotheses begin, however, to appear to the consciousness as a result of the research that has been done so far.

3. *The modern dissemination in print of the* Liber Abaci *and its pitfalls*

In addition to the antique manuscripts, there also undeniably exists, however, a vehicle that, notwithstanding the inadequate and problematic access to the manuscript sources, has spread the text of the Fibonaccian treatise throughout modern and contemporary culture: the well known Italian mathematician and historian of science Baldassarre Boncompagni Ludovisi[11], in fact, in his brilliant far-reaching project which brought into focus the personality of Fibonacci, as well as his surviving works[12], realized and published in Rome in

[9] On the thorny question of the "active tradition" in the transmission of Latin and romance texts, a phenomenon of particular importance from classical times to the middle ages above all in the case of texts of a technical kind, often for personal use of those directly interested, foundational and still useful is the essay by A. Varvaro, *Critica dei testi classica e romanza. Problemi comuni ed esperienze diverse*, «Rendiconti dell'Accademia di archeologia, lettere e belle arti di Napoli», 45 (1970), pp. 73-117; for a more recent analysis of the question, even if limited to scholastic texts, one can refer to the introduction of contribution by F. Bognini, *Tradizioni attive e testi scolastici. Il caso del repertorio «Miramur» di Alberico di Montecassino* in *Prassi ecdotiche. Esperienze editoriali su testi manoscritti e testi a stampa*. Milano, 7 Giugno e 31 ottobre 2007, ed. by A. Cadioli and P. Chiesa, Milano 2008, pp. 95-115.

[10] This investigation, in particular, is carried out by Concetta Carotenuto, who is preparing on the argument her doctoral dissertation (above, n. 6).

[11] Regarding him, V. Cappelletti, *Boncompagni Ludovisi, Baldassarre*, in, *Dizionario biografico degli italiani*, XI, Roma 1969, pp. 704-709; M. Mazzotti, *For science and for the Pope-king: writing the history of the exact sciences in nineteenth-century Rome*, in «British Journal for the History of Science», 33 (2000), pp. 257-282, especially pp. 259-265.

[12] B. Boncompagni, *Della vita e delle opere di Leonardo Pisano, matematico del secolo decimoterzo*, in «Atti dell'Accademia pontificia dei nuovi Lincei», 5 (1851-1852), pp. 208-245; B. Boncompagni, *Intorno ad alcune opere di Leonardo Pisano*, Roma 1854; B. Boncompagni, *Tre scritti inediti di Leonardo Pisano [...] secondo la lezione di un codice della Biblioteca Ambrosiana di Milano*,

1857 what can with ample justification be defined the *editio princeps* of the entire treatise[13].

Boncompagni, however, notwithstanding the legitimacy and seriousness of his intentions, his undoubted abilities as a scholar and his uncommon erudition, realized his editorial task properly with the methods in vogue in his time: with scarce philological awareness and without even bothering to append any sort of comment. He faithfully reproduced the transcription, almost always uncritically however, of a single 14th century codex, which is now preserved in Florence, in the Biblioteca Nazionale Centrale, with the shelf mark Conv. Soppr. C. 1. 2616, ff. 1-214. It of course was a noteworthy editorial operation, especially as it made available in print to a vast number of interested parties a work which had almost fallen into oblivion and that up to that time could be consulted only from its manuscript sources, with all the difficulties and inconvenience which this could entail. Even so, it spread the text of the Fibonaccian treatise in a form and substance which was often misleading as it was offered in only one of the numerous manuscript codices from which it has transmitted to us: this codex, as is confirmed after an initial investigation, and also when compared, even if not completely, with some of the other manuscript sources, not only does not have the merit of transmitting the best texts of the *Liber Abaci*, but includes mistakes or omits passages of various kinds, which make, at times, comprehension problematic.

Needless to say this is the text on which, for more than a century and a half, generations of scholars, most of all historians of science, have labored to add *tesserae* to our understanding of the evolution of the history of mathematics and numerical calculation: how many pitfalls can be hidden in the construction of a complex intellectual framework based on a text whose completeness and authenticity is at best questionable, can be easily understood even by someone lacking the sophisticated specialized knowledge of a philologist[14].

Firenze 1854; B. Boncompagni, *Opuscoli di Leonardo Pisano*, Firenze 1856; B. Boncompagni, *Scritti di Leonardo Pisano, matematico del secolo decimoterzo*, voll. I-II, Roma 1857-1862 (the first volume contains the edition of the *Liber Abaci*, the second the edition of the *Practica Geometriae*, in addition to other minor writings).

[13] B. Boncompagni, *Il* Liber Abbaci *di Leonardo Pisano pubblicato secondo la lezione del codice Magliabechiano C. I. 2616, Badia Fiorentina, n° 73*, in *Scritti di Leonardo Pisano, matematico del secolo decimoterzo*, vol. I, Roma 1857, pp. 1-459. Before this edition, in fact, only the *Prologus* and chapter XV of the *Liber Abaci* had enjoyed a respectful circulation in print due to the work, in particular, of G. Libri, *Histoire des sciences mathématiques en Italie, dépuis la renaissance des lettres, jusqu'à la fin du dixseptième siècle*, T. II, Paris 1838, respectively pp. 287-290 and 307-476. Libri, which in relation to the *Prologus* does not fail to mention two prior 17th century editions, for its own transcription made use of the ms. Magl. XI. 21 of the Biblioteca Nazionale Centrale di Firenze.

[14] That the choice to refer to a single manuscript (Firenze, Biblioteca Nazionale Centrale, Conv. Soppr. C. 1. 2616, ff. 1-214) was the principal weakness of the edition of Boncompagni was already evident to R.E. Grimm, *The autobiography of Leonardo Pisano*, in «The Fibonacci Quarterly», 11 (1973), pp. 99-104.

On the 19th century *editio princeps* of Boncompagni of 1857, which represents, therefore, almost a diplomatic transcription of its only source, is also recently based, notwithstanding the not insignificant problems just discussed, the only complete translation of the *Liber Abaci* so far made into a modern language, edited in English by Lawrence E. Sigler and published in 2002[15]. In an era like our own, which tends increasingly to lose more and more contact with classical languages and in which there are always fewer cultured persons who have mastered the ability to read and understand Latin at a sufficiently high level, the work certainly has the merit of making readily available the Fibonaccian work to a public as wide and international as possible; furthermore it corrects, albeit without giving an explicit account, a good number of calculation errors, which appear in its original – that is in Boncompangni's 1857 edition – on the basis of its manuscript source. Unfortunately, however, it has succeeded in inevitably adding to the already numerous mistakes and problems of its original a series of over-simplifications and misunderstandings, not only because of the insistence on a text which seems suspect for the already adduced reasons, but also because it exhibits in some cases unreliable, if not, at times, bizarre interpretations, owing to the difficulties encountered by a makeshift translator in deciphering a language, the medieval Latin, full of hidden pitfalls especially for a non-specialist. Its writer, in fact, cannot be numbered among Latin language specialists; and certainly not among those of medieval Latin, which in many ways is very distant from the classical variety, and often fraught with difficulties as well due to its too recent tradition of scholarship[16] with respect to the defining of the semantic categories of certain morphemes, lexical items and stylistics.

4. *The need for new editorial treatment of the* Liber Abaci

At present, therefore, our official knowledge of Fibonacci's *Liber Abaci*, apart from the few specialized studies by a handful of mathematicians and historians of science focused on single and for the most part short passages of its Latin text, and even with the help of some other manuscript witnesses which have been transmitted to us[17], has a foundation consisting of only two supporting structures: the first is the unabridged edition of its Latin text edited by Boncompagni (1857) and the second is the English language translation of that text produced by Sigler (2002); neither, as we have already seen, reliable or sound.

[15] L.E. Sigler, *Fibonacci's* Liber Abaci. *A Translation into Modern English of Leonardo Pisano's Book of Calculation*, New York-Berlin-Heidelberg 2002.
[16] On the problems of the medieval Latin language and philology compared to those of classical antiquity, G. Germano, *Quindici anni dopo tra fortuna ed attualità: la Miscellanea di Studi per Alessandro Perosa e lo stato odierno della Filologia Medioevale e Umanistica*, in «Vichiana», 3 (2001), 2, pp. 287-314 and specially pp. 292-295, with the bibliography there cited.
[17] It is the case, for example, of Grimm, *The autobiography* cit.

Although, therefore, the importance of Fibonacci's *Liber Abaci*, as pointed out at the outset of the present essay, has been considered by many to be basic not only in the history of mathematics, but also, in more general terms, of western culture itself, its text, nonetheless, must be judged to have not yet had an editorial treatment either equal to this significance, or which is reliable with respect to current philological methodology. As such it has remained to date of scant and uncertain accessibility for historians of science and other interested scholars. It is of urgent importance therefore, now that it seems interest in the contributions of Fibonacci to the history of science and western culture has reawakened[18], to work seriously to produce a new edition of the *Liber Abaci* of Leonardo Pisano which has a firm foundation from a philological point of view. It is necessary that to all those interested is made available a text of the treatise which, regarding all or most of the surviving tradition[19], is as complete and authentic as possible, so that it provides a new and more reliable point of departure for specialized studies of each discipline.

Such a text, critically reconstructed in relation to, or, in technical terms, on the basis of a collation of all the known manuscript sources[20], should have the status of being as close as possible to the authentic and original form conceived of by the author; it should also introduce, by means of an appropriate apparatus, the significant variants that can be found in all the manuscripts from which it has been transmitted. An accurate and precise translation in a modern language should be based on it; this should be both thoroughly annotated with a view to clarifying problems of textual interpretation – ranging from those of a linguistic character to those more properly considered scientific, while not neglecting the study of mathematical and algorithmic aspects –, as well as appended with other useful aids such as a glossaries and indices.

Finally, it would be desirable, not to say indispensable, that the results of such a work, in our era dominated by instantaneous and total communication in the form of hypertexts made possible by digital and other information technologies, were made extensively usable on the internet, in order to be open to and compatible with, in a more evolved configuration compared to common print-

[18] Reference can be made, even if only as an example, to *Fibonacci tra arte e scienza*, ed. by L.A. Radicati di Brozolo, Pisa 2002; as well as N. Geronimi, *Giochi matematici del Medioevo. I "conigli di Fibonacci" e altri rompicapi liberamente tratti dal* Liber Abaci, with a Preface by P. Nastasi, Milano 2006 (the text presents translations in Italian of single portions of the *Liber Abaci* always reproduced by the *editio princeps* of Boncompagni).

[19] To the up to now known manuscript sources I am sure that the census of the manuscript witnesses of the *Liber Abaci*, that the research group directed by the writer is carrying out, will be able to bring interesting additions, since an investigation of this kind, as I have said above, has never been undertaken.

[20] Given that the tradition of the work, as we have already had occasion to consider, also includes manuscript witnesses in the Tuscan vernacular, it is necessary to take this into account: if they will not consent, as is obvious, to a punctual and continuous fully usable comparison, they will be able, nevertheless, furnish an important contribution in doubtful cases or in the presence of lacunae, once the family of witnesses on which they depend is established.

ed editions, the new possibilities offered by computerization. In this way, the critical text, reproductions of manuscript sources, philological apparatus, translation, commentary and other aids, such as indices and glossaries, could be made easily available and left to the personal choice of every sort of user, from the most desultory and casual to the most sophisticated and exacting; all in hyper textual format projected toward the future of the philological and scientific information world.

5. *Toward a critical edition of the* Liber Abaci

It is understood that the realization of such an editorial project can not be accomplished without a close synergy of diverse specialized skills, which are often very different from each other: precisely for this reason a diversified work group has been constituted which, made up of specialists of different cultural backgrounds – philologists of medieval Latin, historians of science and information scientists –, has already begun to work in *équipe* toward the worthy goal of producing a modern critical edition of the *Liber Abaci* furnished not only with a translation and commentary, but with every other tool useful for the exegesis and dissemination of the Fibonaccian treatise[21].

The cornerstone of the project, however, beyond the application of all the modern instruments and theoretical results of medieval Latin philology, is the goal of making available on-line, on a web site accessible to all interested persons, the critical text of the *Liber Abaci* and its Italian and English translations together with all those aids which can render truly useful to a wider public, both specialists and the merely interested, the Fibonaccian treatise: the reproduction of the manuscript sources, especially relative to the critical sections where the principal textual variants have been identified; the utilization of simple animations to clarify the calculation processes explained by the author of the treatise and illustrated by him in an understandable, but fixed form by means of the numerous tables which appear throughout all the manuscript tradition in the margins of the corresponding pages; the introduction of short films that can make more familiar and understandable the devices (or, in our modern terminology, the algorithms) proposed by Fibonacci to solve the problems he addresses – all objectives attainable, naturally, through links to portions of the corresponding text –, these should be the salient features of such a digital edition.

At present a prototype of a web site has been created in experimental form[22] that contains provisional results of the work completed on only the first chap-

[21] The first results of this cooperative effort are merged in Burattini, *Per un'edizione* cit., pp. 55-138.

[22] A detailed description of this prototype, with a presentation of its principal constituent characteristics and its encoding modality, has been prepared by E. Burattini, L. Sauro, *Verso un'edizione digitale* on-line *del* Liber Abaci, in Burattini, *Per un'edizione* cit., pp. 102-105. The site can currently be viewed at url <http://people.na.infn.it/~sauro/fibonacci/LiberAbaci/liberAbaci.html>,

ter of the *Liber Abaci*. However changes are envisioned in the near future which will account for all results to date going far beyond the first chapter of the treatise and which will be designed to facilitate a form of interaction between the authors and a platform for subscribed users. The latter will be able to collaborate directly to the perfecting and integration of what has been submitted to the network through a *forum* installed for this purpose. Obviously, this is not a unique experience in the field of philological studies[23], but it would be one of the few occasions of interdisciplinary collaboration between experts in the humanities and experts in the scientific-technological disciplines, which involves the scientific content of the web site and not just the technical process of producing and publishing it. A true proof of the value of interdisciplinarity, this project could go far toward clearly and irrefutably demonstrating both how much and what the too often criticized disciplines of the humanities can contribute to the advancement of scientific knowledge through the recovery of its historical roots.

6. *A first editorial essay: the dedicatory* Epistula *to Michael Scotus and the autobiographical* Prologus *of the* Liber Abaci

6. 1. *Prefatory note to the critical text*

For its great historical and cultural importance, universally recognized by specialists of multiple and various research sectors, it seems opportune to republish in this space the critical edition of the introduction to Leonardo Pisano's *Liber Abaci*, which is represented by the diptych composed of a dedicatory *Epistula* of the treatise to Michael Scotus, a well known philosopher and astrologer at the court of the emperor Frederick II of Swabia[24], and of an autobiographical *Prologus*. To his already critically constituted text[25], which here, however, is reviewed in the light of the recent acquisition of the Berlin manuscript (see above) and of a more detailed collation of the previously known manuscript witnesses, I will follow in this venue with a new translation in English[26].

but it cannot be reached from search engines and will be soon be moved elsewhere in expectation of its full functionality.

[23] Reference can be made, only as an example, to the French experience with the digital edition of the Donatus' *Commentary* on Terence's *Comedies*: *Hyperdonat, une édition électronique des commentaires de Donat aux comédies de Térence. Bruno Bureau, Maud Ingarao, Christian Nicolas, Emmanuelle Raymond (dir.), CEROR, Université Lyon III, ENS de Lyon, 2007-2011*, <http://hyperdonat.ens-lyon.fr>.

[24] Regarding him and his relationship with Fibonacci, Caianiello, *La vita e l'opera* cit., § 2.1, pp. 59-65 and note 26; § 2.2, pp. 65-72 and notes 31-33; as well as the *Appendice I*, pp. 109-112.

[25] G. Germano, *Appendice II*, in Burattini, *Per un'edizione* cit., pp. 121-125. For the ecdotic criteria see *ibidem*, pp. 117-120.

[26] For an useful commentary on this text refer to C. Carotenuto and E. Caianiello, *Commento* in *Appendice II*, in Burattini, *Per un'edizione* cit., pp. 126-138.

This introduction to the Fibonaccian treatise, even if not always in its entirety, has been transmitted to us, at least as far as we currently know, by the manuscript witnesses here listed:

- *B* Berlin, Staatsbibliothek, Preußischer Kulturbesitz, ms. Lat. Fol. 418, paper, 17th century, ff. 1*r*-3*v*
- *F* Firenze, Biblioteca Nazionale Centrale, ms. Conv. Soppr. C. 1. 2616, parchment, 14th century, f. 1*r*
- F_2 Firenze, Biblioteca Nazionale Centrale, ms. Fond. Prin. II. III. 25 [Magl. XI. 22], paper, 15th or 16th century, f. 1*r*
- F_1 Firenze, Biblioteca Nazionale Centrale, ms. Magl. XI. 21, parchment, 14th century, f. 1*r*
- *R* Firenze, Biblioteca Riccardiana, ms. 783, paper, 15th century, ff. 1*r*-*v*
- *A* Milano, Biblioteca Ambrosiana, ms. I. 72 Sup., parchment, 13th century, f. 1*r*
- *N* Napoli, Biblioteca Nazionale, ms. VIII. C. 18, paper, 17th century, ff. 3*r*-4*r*
- *S* Siena, Biblioteca Comunale degli Intronati, ms. L. IV. 20, parchment, ending 13th - early 14th century, f. 1*r*
- *V* Città del Vaticano, Biblioteca Apostolica Vaticana, ms. Palat. Lat. 1343, parchment, ending 13th - early 14th century, f. 1*r*

Of these manuscript witnesses[27] two, *A* and F_2, are missing the dedicatory *Epistula* to Michael Scotus and one, *S*, is missing the autobiographical *Prologus*. Moreover, in *B* the dedicatory *Epistula* is written after the *Prologus*; whereas in F_1 the *Epistula* appears to have been added at a later time and written with a smaller body size, but by the same hand – at least it seems so – as the rest of the manuscript copy. This *Epistula* is here located in the right hand margin of f. 1*r* and the framework text displays only the autobiographical *Prologus* just after the *incipit*. Finally, in F_2, f. 1*r*, the *Prologus* is transmitted in a version written in the Tuscan vernacular, unlike the rest of the manuscript, which, except that for the first lines of the first chapter, displays the Fibonaccian text in its original Latin form.

Even if the history of the writing of the *Liber Abaci*, as is well-known, takes into account two different editions by the author, the first from 1202 and the second from 1228, of which only the latter was dedicated to Michael Scotus[28], the absence of the dedicatory *Epistula* in *A* and in F_2, as well as in the framework text of F_1, does not seem to mean at all, however, that these manuscripts are necessarily witnesses of the oldest drafts of the treatise: since from what is revealed by the text collation of these with the other complete manuscripts that so far has been completed – and that comprises the first six chapters – there does not seem to be among them any variants which could be judged to be due to an editorial work. Furthermore, since the only draft of the Fibonaccian treatise which we have, at least in light of the partial results of our investigations

[27] The here mentioned manuscripts have been subjected to a systematic, although at the moment only partial, process of collation: this operation, within the team working on the edition of the *Liber Abaci*, is entrusted to the care of Concetta Carotenuto, who is preparing, as I have already said, her doctoral dissertation on Fibonacci's *Liber Abaci* (above, n. 6).

[28] Caianiello, *La vita e l'opera* cit., § 2.2, pp. 65-72.

to date, would seem to be the definitive 1228 edition, the irregularities in the *Epistula*'s and *Prologus'* tradition in all likelihood should not be attributed to editorial factors. It isn't reasonable to assume, moreover, that tradition's problems of such a kind can be always indicative of a stemmatic relationship, because, at least from what it seems to stand out in the light of the collation, which has been accomplished so far, the nine manuscripts in question can be grouped into two distinct families: *F* F_2 *R S* belong to the first and *A B* F_1 *N V* belong to the second. Thus, *A* and F_2, which lack the *Epistula*, do not belong to the same branch of the tradition; while the fact that in *B* the dedicatory *Epistula* has been written after the *Prologus* could be connected – but we can't yet say with certainty how – with the particular *facies* exhibited by F_1, where the *Epistula* seems added at a later time in the right hand margin of f. 1*r*[29].

As to the particular condition of F_2, which presents the *Prologus* in a vernacular version, its text cannot be used for the purpose of establishing a critical text unless, as I believe, in a single case, where we find in F_2 the lection *arte di Pittagora*, which corresponds to the lection *arcus Pictagore* (§ 3), transmitted by *F R*, as opposed to the lection *Pictagore* transmitted by *A B* F_1 *N V* and clearly presenting an omission. This lection should require *artem Pictagore* in its original Latin text, as proof of an understandable trivialization in the tradition of the text, either due to the ignorance of the copyist (who didn't understand any more what had been meant by *arcus Pictagore*) or a confusion deriving from paleographic reasons. This confusion occurred in the modern era as well in specialized discussions of the issue[30].

In establishing the critical text, care has been taken above all, in the absence of autographic witnesses and also to achieve a simpler and more accessible usability, to conform both a certain number of graphic oscillations (for example: *abacus*/*abbacus*, *practica*/*pratica*, etc.) to the more prevalent form in the area of its specific tradition and/or of the medieval Latin[31]; and, as well, to conform it to modern punctuation and to modern scansion in paragraphs of the single composition units.

As regards the general orthographic appearance of the critical text, a series of small changes have been effected by tacit agreement without giving any acknowledgement in the apparatus' notes: for example, the abbreviations and the tachygraphy compendia, in great numbers in the manuscript witnesses that we

[29] On the top of the front endpaper *verso* of the manuscript *B* a hand, that seems to belong to the early 20th century, wrote the following note: «Copié sur le Ms. Classe XI, N°21 de la Bibl. Magliabechiana de Florence»; but at the current status of research we aren't yet in a position either to confirm or to refute with certainty its content, which would seem to assert a direct dependence on F_1.

[30] Grimm, *The autobiography* cit., pp. 99-100.

[31] As far as the graphic confusion of the nexus *-ti-* with that *-ci-* and viceversa – very common in writing medieval practice on the basis of a certain phonetic identity – the circumstance of absolute and ungovernable whim present in the manuscript tradition has convinced us of the need for a normalization with the classical language.

have, have been eliminated; the often whimsical use of capital and small letters has been normalized; the graphic *v* in place of *u* with consonant value has been adopted, and the same for *i* in place of *j*; however, the custom of expressing the antique diphthong *ae* with the simple graphic *e* has been preserved, as well as other graphic particulars of minor importance, inasmuch as they are characteristic of medieval linguistic practice.

As regards punctuation, the modernization introduced according to a criterion almost universally used in the editing of medieval texts aims above all at allowing an easier and more readily responsive logical-syntactic articulation of the text in light of our reading sensibilities; but it also takes into account the indications, anything but irrational, already present in the manuscripts. Finally, the numerical scansion in paragraphs of the single composition units of the text, which has been included above all for rapid, unambiguous citation, also has the purpose of indicating unequivocally the correspondence between the Latin text either with the notes of critical apparatus or with its English language translation.

As regards the other observed ecdotic criteria, it has been possible, on the basis of tradition's errors of an extensive enough portion of text, to group with a certain degree of reliability the transmitted manuscript witnesses, as I have said above, only into two distinct families. Whether because the investigation as a whole is still in its initial stages, since the collation of the manuscripts, as we have seen, is still only partially complete, or whether because the manuscript witnesses within these families seem to be all independent of one another, but also because some errors appear to be independent from each other (polygenesis), consequently the choice of the critical lection, when not confronting one of the many obvious blunders present in some of the single representatives of the tradition, has been made often on the basis of the *lectio difficilior*, of the medieval language *standard,* or of the intelligence of the text in light of the current level of knowledge of the science history[32].

The header of the dedicatory *Epistula* to Michael Scotus is the result of a conjectural critical reconstruction aimed at restore for this text portion its effective epistolary character: this operation, which is based in large part on the lection of *N*, however, draws on other elements widely attested in the rest of the tradition. Such a critical choice finds its meaning also in the desire to simplify and adapt to modern sensibilities the heavy opening formula which appear in most of the witnesses.

Also the <*Prologus*> title of the autobiographical *Prologus* section has been critically integrated, but it finds its more than rational justification in the *explicit* of the same section (*Explicit prologus*), present in *A B F* F_1 *R*.

[32] Particularly insidious, for example, is the variantistic *facies* of *N*, which seems to have a tendency to normalize certain expressions characteristic of medieval Latin according to the *standard* of classical Latin (cfr. *Epistula*, § 1 *Scripsisti* for *Scripsistis*; *indagatione* for *indagine*; *Prologus*, § 1 *me in* for *ibi me*; § 2 *incubui* for *intellexi*).

In *Prologus* § 3 the word *Algorismus* was placed in italics, to make clear that the author refers to Latin adaptations of the lost al-Khwārizmī text (9th century) based on the Indo-Arabic number system[33].

The critically established text of the dedicatory *Epistula* and of the *Prologus* has been equipped with a positive critical apparatus, which, however, provides confirmation of the individual manuscript witnesses only for the rejected variants.

6. 2. *The critically established text and the apparatus of the variants*

Leonardus filiorum Bonaccii Pisanus Michaeli Scotto summo philosopho.

(1) Scripsistis mihi, domine mi et magister Michael Scotte, summe philosophe, ut librum de numero, quem dudum composui, vobis transcriberem: unde, vestre obsecundans postulationi, ipsum subtiliori perscrutans indagine ad vestrum honorem et aliorum multorum utilitatem correxi. In cuius correctione quedam necessaria addidi et quedam superflua resecavi. (2) In quo plenam numerorum doctrinam edidi iuxta modum Indorum, quem modum in ipsa scientia prestantiorem elegi. (3) Et quia arismetica et geometrica scientia sunt connexe et suffragatorie sibi ad invicem, non potest de numero plena tradi doctrina, nisi interserantur geometrica quedam, vel ad geometriam spectantia, que hic tantum iuxta modum numeri operantur, qui modus est sumptus ex multis probationibus et demonstrationibus, que figuris geometricis fiunt. (4) Verum in alio libro, quem de practica geometrie composui, ea que ad geometriam pertinent et alia plura copiosius explicavi, singula figuris et probationibus geometricis demostrando.

(5) Sane hic liber magis quam ad theoricam spectat ad practicam, unde qui per eum huius scientie practicam bene scire voluerint, oportet eos continuo usu et exercitio diuturno in eius practicis perstudere, quod, scientia per practicam versa in habitum, memoria et intellectus adeo concordent cum manibus et figuris, quod quasi uno impulsu et anhelitu in uno et eodem instanti circa idem per omnia naturaliter consonent: et tunc cum fuerit discipulus habitudinem consecutus, gradatim poterit ad perfectionem huius facile pervenire.

(6) Et ut facilior pateret doctrina, hunc librum per quindecim distinxi capitula, ut quicquid de his lector voluerit, possit levius invenire. (7) Porro sin in hoc opere reperitur insufficientia vel defectus, illud emendationi vestre subicio.

<*Prologus*>

(1) Cum genitor meus a patria publicus scriba in duana Bugee pro Pisanis mercatoribus ad eam confluentibus constitutus preesset, me in pueritia mea ad

[33] C. Carotenuto and E. Caianiello, *Commento* in *Appendice II*, in Burattini, *Per un'edizione* cit., pp. 136-138.

se venire faciens, inspecta utilitate et commoditate futura, ibi me studio abaci per aliquot dies stare voluit et doceri. (2) Ubi ex mirabili magisterio in arte per novem figuras Indorum introductus, scientia artis in tantum mihi pre ceteris placuit et intellexi ad illam, quod quicquid studebatur ex ea apud Egyptum, Syriam, Greciam, Siciliam et Provinciam cum suis variis modis, ad que loca negotiationis causa postea peragravi, per multum studium et disputationis didici conflictum.

(3) Sed hoc totum etiam et *Algorismum* atque arcus Pictagore quasi errorem computavi respectu modi Indorum. (4) Quare amplectens strictius ipsum modum Indorum et attentius studens in eo, ex proprio sensu quedam addens et quedam etiam ex subtilitatibus Euclidis geometrice artis apponens, summam huius libri, quam intellegibilius potui, in quindecim capitulis distinctam componere laboravi, fere omnia que inserui certa probatione ostendens, ut ex ea perfecto pre ceteris modo hanc scientiam appetentes instruantur et gens Latina de cetero sicut hactenus absque illa minime inveniatur.

(5) Si quid forte minus aut plus iusto vel necessario intermisi, mihi deprecor indulgeatur, cum nemo sit qui vitio careat et in omnibus undique sit circumspectus.

Leonardus – philosopho] Incipit liber Abaci compositus a Leonardo filio Bonacii Pisano in anno MCCII *F* F_1, Incipit liber abbaci compositus a leonardo filiorum bonaccii pysano in anno MCCII et correctus ab eodem XXVIII *R*, Leonardus filius Bonaccii Pisanii Michaeli Scotto summo philosopho *N*, Incipit abbacus Leonardi de domo filiorum bonacii pisanii compositus a. MCCII et correptus ab eodem a. MCCXXVIII *S*, *deest in V*; *Epistula ad Michaelem Scottum, quam B post* Prologum *et* F_1 *in mg. dx. tradunt, deest in A* F_2

(1) Scripsistis] Scripsisti *N* mi et magister] mi magister *F* postulationi ipsum] postulationes *R* perscrutans] prescrutans *R S*, prescriptans F_1 indagine] indagatione *N* (2) plenam] planam *V* in ipsa – prestantiorem] prestantiorem in ipsa scientia *R*, in ipsa scientia aliis prestantiorem *S* *(3)* quia] que *F* F_1, quem *V* arismetica] arismetrica *B F* F_1 *R*, arithmetrica *N* geometrica scientia] geometria scientia *F V*, geometrie scientia *B* F_1 suffragatorie] *ex* subflagratorie *corr. in R* interserantur] intersecantur *F* F_1 *R* tantum] tamen *B* (4) practica] patrica F_1 et alia plura] *om. R* copiosius] copiosis *B F* F_1 singula – probationibus] singula subiectis approbationibus *F*, singulis figuris et probationibus *R* demostrando] *om. B* (5) magis – praticam] ad practicam magis quam ad theoricam spectat *N*, magis ad theoricam spectat quam ad practicam *F* voluerint] *ex* voluerunt *corr.* in *R* continuo] continue *F* perstudere] praestudere *N* intellectus] intellectu *R* figuris … habitudinem …poterit] signis …latitudinem … poteris *B* huius facile] huius scientie facile *S* (6) pateret] patiat *R*, pateat *V* ut] unde *B* lector voluerit] voluerit lector *R* *(7)* sin in] si in *B F R V* emendationi – subicio] emendationi subicio *R*, emendationi vestre subicio. Explicit prologus incipiunt capitula *S*, ad emendationi vestre subicio *V*

<Prologus>] Incipit liber Abaci compositus a Leonardo filio Bonaccii Pisano in anno MCCII *B N*, Incipit liber abbaci compositus a Leonardo filiorum bonacii pisano in anno MCCII et correctus ab eodem XXVIII *A V*, *quamdam textus continuitatem nec ullum titulum exhibent F R*, Traductione. / Incomincia un libro di Abbaco composto da Leonardo Pisano nell'anno MCCII e corretto dal medesimo nel 28 F_2, qui *italice versum* Prologum *exhibet*, *omnis* Prologus *deest in S*

(1) eam] eum *A B V* ibi me] me in *N* stare] instrui *N*, ita esse *B*, ita est F_1 (2) arte] artem *V* intellexi ... suis] incubui ... supradictis *N* causa] tam *F* postea] prius ea *B*, *om*. *R* (3) etiam et] etiam *R*, etiam et ad *V* atque] adque ***V*** arcus] arte F_2, *om*. *A B* F_1 *N V* (4) Indorum et] Indorum etiam *R* eo]eum *B* geometrice] geometrie *A B* F_1 *N* artis] arctis *R* summam] summa *N* capitulis] capitus *V* distinctam] distincta *A* F_1 *N V* ex ea] ex causa *B*, ex tam *R*, extra *A F V* perfecto] perfecta *B*, perfectam F_1 *N* pre] pro *B* (*ex* pre *fortasse corr.*) inveniatur] inveniantur *R* deprecor] de pecto *N* *(5)* circumspectus] circumspectus. Explicit prologus. Incipiunt capitula *A F R*, circumspectus. Explicit Prologus. Incipit capitulum *B* F_1

6. 3. *Translation of the critically established text*

Leonardo Pisano Fibonacci to the supreme philosopher Michael Scotus

(1) You have written to me, my lord and master Michael Scotus, supreme philosopher, to send to you a copy of the book on numbers that some time past I had written: to accomplish this, in order to satisfy your request, I have subjected it to a rather careful revision in your honor and for the use of many others. And in correcting it I have added some necessary notions and eliminated some superfluous passages. (2) In this book I have published the entire doctrine of numbers according to the method of the Indians, a method that I have adopted in this same science as it is the most effective. (3) And since the science of arithmetic and geometry are linked and bear each other out, the doctrine of numbers cannot be taught in its entirety, if not with the aid of certain geometric principles, or of notions which pertain to geometry, that are applied in this area only by means of the numbers method, a method that has been established with many proofs and demonstrations that are carried out using geometric figures. (4) However in another book that I wrote on the practice of geometry, I explained with a greater wealth of detail the principles belonging to geometry as well as many others, demonstrating them one by one with geometric figures and proofs.

(5) Certainly, this book is concerned more with practice than with theory and for this reason those that wish with his help to know well the practice of this science of numbers, it is necessary that they dedicate themselves with much diligence to its applications with continuous use and long exercise, so that, once they have transformed theoretical knowledge into habit by means of practice, memory and intellect insomuch are in agreement with hands and figures, that they work harmoniously toward a single purpose with the help of all possible means as if with a single impulse and yearning in the same and single instant: and only when the disciple will have acquired the habit, step by step, can he easily reach at the perfect attainment of this practice.

(6) And so that the doctrine can be understood more easily, I have divided this book into fifteen chapters, so that the reader can find the topic he seeks among these with greater speed. (7) But if instead it is found in this work a deficiency or a shortcoming, I submit it for your correction.

Prologue

(1) When my father was nominated by the homeland a public customs official of Bejaïa and was responsible in the name of the merchants of Pisa who flocked there, he got me come with him when I was child and, considering the use and the advantage that would derive from this, he wanted me to stay there for some time and that I be instructed in the study of the abacus. (2) Having been introduced there to this art with an amazing method of teaching by means of the nine figures of the Indians, I loved the knowledge of such an art to such an extent above all other arts and so much did I devote myself to it with my intellect, that I learned with very earnest application and through the technique of contradiction anything to be studied concerning it and its various methods used in Egypt, in Syria, in Greece, in Sicily, and in Provence, places I have later visited for the purpose of commerce.

(3) But all this and even the *Algorism* and the *Arcs of Pythagoras* I considered almost erroneous compared to the method of the Indians. (4) For this reason, agreeing more scrupulously to the method of the Indians and applying myself with greater attention to it, not without adding something of my own thought and not without adopting also some principles taken from the subtleties of the geometric art of Euclid, I forced myself to compose, in the most understandable way I could, the summa of this book divided into fifteen chapters. I demonstrated with certain proof almost everything I placed in it, in order that – the book having been perfected beyond all others as a consequence of this trial – those that now aspire to this knowledge can be instructed in it, and also so that the Latin people do not find themselves unfamiliar with it, as has been the case until now.

(5) If by chance I have neglected less or more than just or necessary, I beg that I can be forgiven, since there is no one without flaws no matter how much in all and everywhere prudent.

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